FULLY WORKED SOLUTIONS

Context 9: The birth of modern physics

Chapter 21: From plum puddings to Schrödinger's cat

Chapter questions

1. The fourth spectral line in the Balmer series corresponds to values of $n_f = 2$ and $n_i = 6$.

$$
\begin{aligned}
\frac{1}{\lambda} &= R_H\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \\
&= 1.097 \times 10^7\left(\frac{1}{2^2} - \frac{1}{6^2}\right) \\
&= 1.097 \times 10^7\left(\frac{1}{4} - \frac{1}{36}\right) \\
&= 2.44 \times 10^6 \\
\lambda &= 4.10 \times 10^{-7}\ \text{m} \\
&= 410\ \text{nm}
\end{aligned}
$$

2.

$$
\begin{aligned}
\frac{1}{\lambda} &= R_H\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \\
&= 1.097 \times 10^7\left(\frac{1}{1^2} - \frac{1}{5^2}\right) \\
&= 1.097 \times 10^7\left(1 - \frac{1}{25}\right) \\
&= 1.05 \times 10^7 \\
\lambda &= 9.5 \times 10^{-8}\ \text{m} \\
&= 95\ \text{nm}
\end{aligned}
$$

3. $$\frac{1}{\lambda} = R_H\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$\frac{1}{3.89\times10^{-7}} = 1.097\times10^{7}\left(\frac{1}{2^2} - \frac{1}{x^2}\right)$$

$$= 1.097\times10^{7}\left(\frac{1}{4} - \frac{1}{x^2}\right)$$

$$\frac{1}{3.89\times10^{-7}\times1.097\times10^{7}} = \frac{1}{4} - \frac{1}{x^2}$$

$$\frac{1}{x^2} = \frac{1}{4} - \frac{1}{3.89\times10^{-7}\times1.097\times10^{7}}$$

$$= 1.566\times10^{-2}$$

$$x^2 = 63.9$$

$$x = 8$$

This line corresponds to an value of n_i= 8, therefore it is the sixth line in the Balmer series.

4. $r_n = n^2 \times 5.3\times10^{-11}$

$$r_5 = 5^2 \times 5.3\times10^{-11}$$
$$= 1.3\times10^{-9}\text{ m}$$

5. (a) If an electron is to be removed from the hydrogen atom, its total energy must be increased to zero.

In its sixth excited state, which corresponds to a value of n= 7, the energy of the electron will be:

$$E_n = -\frac{13.6}{n^2}$$
$$E_7 = -\frac{13.6}{7^2}$$
$$= -0.278\text{ eV}$$

It will therefore require 0.278 eV to remove the electron from the atom.

(b) 0.278 eV is equal to $0.278 \times 1.6 \times 10^{-19}$ J = 4.45×10^{-20} J

6. The third and seventh excited states correspond to the states for n=4 and n=8.

The energy difference between these states is:

$$\Delta E = \frac{E_1}{4^2} - \frac{E_1}{8^2}$$
$$= -13.6(\frac{1}{4^2} - \frac{1}{8^2})$$
$$= -0.64 \text{ eV}$$

The energy difference is 0.64 eV.

7. Transition from n=4 to n=2:

$$\Delta E = \frac{E_1}{2^2} - \frac{E_1}{4^2}$$
$$= -13.6(\frac{1}{2^2} - \frac{1}{4^2})$$
$$= -2.55 \text{ eV}$$

Energy of photon emitted = $2.55 \times 1.6 \times 10^{-19}$ J = 4.08×10^{-19} J

$$E = \mathrm{h}f$$
$$= \frac{\mathrm{hc}}{\lambda}$$
$$\lambda = \frac{\mathrm{hc}}{E}$$
$$= \frac{6.626 \times 10^{-34} \times 3.00 \times 10^{8}}{4.08 \times 10^{-19}}$$
$$= 4.87 \times 10^{-7} \text{ m}$$
$$= 487 \text{ nm}$$

Wavelength of emitted photon = 487 nm

Transition from n=2 to n=1:

$$\Delta E = \frac{E_1}{1^2} - \frac{E_1}{2^2}$$
$$= -13.6(1 - \frac{1}{2^2})$$
$$= -10.2 \text{ eV}$$

Energy of photon emitted = $10.2 \times 1.6 \times 10^{-19}$ J = 1.62×10^{-18} J

$$\begin{aligned} E &= hf \\ &= \frac{hc}{\lambda} \\ \lambda &= \frac{hc}{E} \\ &= \frac{6.626\times10^{-34}\times3.00\times10^{8}}{1.62\times10^{-18}} \\ &= 1.22\times10^{-7}\text{ m} \\ &= 122\text{ nm} \end{aligned}$$

8. The energy of the ground state is –13.6 eV. The energy required to ionise the atom is hence 13.6 eV.

$$\begin{aligned} \lambda &= \frac{hc}{E} \\ &= \frac{6.626\times10^{-34}\times3.00\times10^{8}}{13.6\times1.6\times10^{-19}} \\ &= 9.14\times10^{-8}\text{ m} \\ &= 91.4\text{ nm} \end{aligned}$$

The wavelength of the photon required is 91.4 nm.

9. The Pfund series corresponds to electrons jumping in to the state $n = 5$. The fourth line in this series will correspond to a jump from the state $n = 9$.

$$\begin{aligned} \Delta E &= \frac{E_1}{5^2} - \frac{E_1}{9^2} \\ &= -13.6\left(\frac{1}{5^2} - \frac{1}{9^2}\right) \\ &= -0.376\text{ eV} \end{aligned}$$

$$\begin{aligned} E &= hf \\ f &= \frac{E}{h} \\ &= \frac{0.376\times1.6\times10^{-19}}{6.626\times10^{-34}} \\ &= 9.08\times10^{13}\text{ Hz} \end{aligned}$$

The frequency is 9.08×10^{13} Hz.

$$c = f\lambda$$

$$\lambda = \frac{c}{f}$$

$$= \frac{3.00 \times 10^{8}}{9.08 \times 10^{13}}$$

$$= 3.3 \times 10^{-6} \text{ m}$$

The wavelength is 3.3×10^{-6} m.

10. (a) $hf = W$

$$f = \frac{W}{h}$$

$$= \frac{2.32 \times 1.6 \times 10^{-19}}{6.626 \times 10^{-34}}$$

$$= 5.6 \times 10^{14} \text{ Hz}$$

(b)

$$\text{Energy of photon} = \frac{hc}{\lambda}$$

$$= \frac{6.626 \times 10^{-34} \times 3.00 \times 10^{8}}{3.6 \times 10^{-7}}$$

$$= 5.52 \times 10^{-19} \text{ J}$$

Work function = 2.32 eV.

Energy of most energetic electrons $= 5.52\times10^{-19} - 2.32 \times 1.6 \times 10^{-19}$

$= 1.8 \times 10^{-19}$ J

11. (a)

$$\lambda = \frac{c}{f}$$

$$= \frac{3.00 \times 10^{8}}{9.7 \times 10^{14}}$$

$$= 3.09 \times 10^{-7} \text{ m}$$

$$= 309 \text{ nm}$$

(b)

$$W = hf$$

$$= 6.626 \times 10^{-34} \times 9.7 \times 10^{14}$$

$$= 6.43 \times 10^{-19} \text{ J}$$

12. The highest energy electrons have velocity of 6.3×10^5 m s^{-1}. Hence their kinetic energy is:

$$\begin{aligned} E_k &= \frac{1}{2} mv^2 \\ &= \frac{1}{2} \times 9.1 \times 10^{-31} (6.3 \times 10^5)^2 \\ &= 1.81 \times 10^{-19} \text{ J} \end{aligned}$$

Energy of incident photons:

$$\begin{aligned} E &= \text{h}f \\ &= \frac{\text{hc}}{\lambda} \\ &= \frac{6.626 \times 10^{-34} \times 3.00 \times 10^8}{400 \times 10^{-9}} \\ &= 4.97 \times 10^{-19} \text{ J} \end{aligned}$$

Work function = energy of incident photons – energy of most energetic electrons

$$= 4.97 \times 10^{-19} - 1.81 \times 10^{-19}$$

$$= 3.16 \times 10^{-19} \text{ J}$$

$$= \frac{3.16 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV}$$

$$= 1.98 \text{ eV}$$

13. (a)

$$\begin{aligned} \lambda &= \frac{\text{h}}{mv} \\ &= \frac{6.626 \times 10^{-34}}{8 \times 10} \\ &= 8.3 \times 10^{-36} \text{ m} \end{aligned}$$

(b) The diameter of a nucleon is of the order of 10^{-15} m. The wavelength of the bowling ball is another 20 orders of magnitude smaller than the nucleon! It will not exhibit any wave nature.

14. $\lambda = \dfrac{\text{h}}{mv}$

$$mv = \frac{h}{\lambda}$$
$$= \frac{6.626 \times 10^{-34}}{5.0 \times 10^{-12}}$$
$$= 1.3 \times 10^{-22} \text{ kg m s}^{-1}$$

A particle with momentum of 1.3×10^{-22} kg m s^{-1} has a wavelength of 5.0×10^{-12} m.

15.

$$\lambda = \frac{h}{mv}$$
$$= \frac{6.626 \times 10^{-34}}{2.0 \times 10^{-3}}$$
$$= 3.3 \times 10^{-31} \text{ m}$$

A particle with momentum of 2.0×10^{-3} kg m s^{-1} has a wavelength of 3.3×10^{-31} m.

Review questions

1. In a wave model of light, energy from the light waves striking the surface of the metal would be distributed among all the electrons near the surface of the metal. It would take a considerable interval of time before any of these electrons could acquire sufficient energy to escape from the surface of the metal.

   In a particle model, a single particle (photon) undergoes an interaction with a single electron and passes all of its energy to that electron. If the energy of the photon is sufficient, the very first photon to strike the metal could possibly cause the emission of an electron.

2. The only observation that can be satisfactorily explained by both the wave model and the particle model is that, if the light produces electron emission, increasing the intensity of the light will increase the number of electrons emitted.

3. A photon colliding with an electron will pass all of its energy on to the electron. If that energy is greater than the work function of the metal, the amount of energy required to remove an electron from its surface, the electron may be emitted. As the energy of a photon is related to its frequency ($E=hf$), photons

with a frequency below the threshold frequency will have insufficient energy to bring about the emission of an electron while photons with higher frequency may cause electron emission.

4. (a) De Broglie predicted that matter had a wave nature.

   (b) He suggested that it should be possible to observe the diffraction of electrons from the surface of a crystal. This would confirm his theory by demonstrating the wave nature of electrons.

5. As $\lambda = \frac{h}{mv}$, the electron (which has a smaller mass) will have the longer wavelength.

6. There are 10 possible spectral lines that may be produced. (These correspond to four inward jumps from the state $n = 5$, three from the state $n = 4$, two from the state $n = 3$ and one from the state $n = 2$.)

7. (a) An emission line is produced when an electron jumps inwards; an absorption line is produced when an electron jumps outwards. The photon emitted in the production of an emission line must have exactly the same energy as that of the photon absorbed in producing an absorption line associated with the same two energy levels.

   (b) An absorption spectrum is produced when white light is passed through a cool gas. In this gas it is expected that the electrons are in low energy states. After an electron has been excited to a high energy state (by absorbing a photon), it will immediately jump back to a lower energy state emitting a photon. All absorption lines will therefore involve a low energy state and a higher energy state but will not involve two high-energy states. (You would not have a transition from $n = 4$ to $n = 5$ as there will be no electrons in the $n = 4$ state.) However, there is no problem with an electron that has been excited to $n = 5$ jumping inwards to $n = 4$ producing an emission line.

8. The electrons will return to lower energy states by emitting photons but they will emit photons in any direction. A few may be emitted into the original beam of white light but most will be emitted in other directions.

9. Balmer's equation is an empirical equation. Balmer had no idea what was going on in gases to produce the spectrum. The important thing about the equation developed by Bohr is that it is based on his model of the atom.

Bohr's postulates are the key to describing his model of the atom and to explaining the reason for the production of spectral lines.

10. The line spectrum of hydrogen supports this. The four visible lines in the spectrum of hydrogen are produced by electrons jumping between discrete energy levels. If the energies of electrons in hydrogen atoms were not discrete but could have any value, a continuous spectrum rather than a line spectrum would be produced.

11. Bohr's quantisation condition involves the angular momentum of electrons in the hydrogen atom being quantised. This in turn imposed the restriction on the energies of the electrons in stationary states. If this restriction did not exist, and the rules of classical mechanics were obeyed, there would be a continuous spectrum as there could be no restriction on the energies of the electrons in the atoms.

12. (a) If the two lines correspond to adjacent transitions of the electron (say from E_4 to E_3 and from E_3 to E_2) there will be a higher energy transition from E_4 to E_2. The electron transition from E_4 to E_2 will have energy equal to the sum of the two energies, hence the frequency of the photon emitted must be the sum of the two frequencies.

(b) $f = (2.7 \times 10^{14}) + (4.6 \times 10^{14}) = 7.3 \times 10^{14}$ Hz

13. While the Bohr model of the hydrogen atom was a major step forward, there were problems with Bohr model. How could you have a model that was a mixture of classical and quantum physics? It could not explain a number of observations such as the Zeeman effect, hyperfine lines or the relative intensities of spectral lines. Heisenberg developed quantum mechanics, which was able to explain these observations, and removed any association with classical physics.

14. Bohr realised that his model was limited and was pleased when a new model based entirely on quantum mechanics was developed.

15. (a)

$$\begin{aligned} E &= hf \\ &= 6.63 \times 10^{-34} \times 4.57 \times 10^{14} \\ &= 3.03 \times 10^{-19} \text{ J} \\ &= \frac{3.03 \times 10^{-19}}{1.60 \times 10^{-19}} \\ &= 1.89 \text{ eV} \end{aligned}$$

(b)

$$\begin{aligned} E &= hf \\ &= 6.63 \times 10^{-34} \times 2.47 \times 10^{14} \\ &= 1.64 \times 10^{-19}\ \text{J} \\ &= \frac{1.64 \times 10^{-19}}{1.60 \times 10^{-19}} \\ &= 1.02\ \text{eV} \end{aligned}$$

(c)

$$\begin{aligned} E &= hf \\ &= \frac{hc}{\lambda} \\ &= \frac{6.63 \times 10^{-34} \times 3.00 \times 10^{8}}{632 \times 10^{-9}} \\ &= 3.15 \times 10^{-19}\ \text{J} \\ &= \frac{3.15 \times 10^{-19}}{1.60 \times 10^{-19}} \\ &= 1.97\ \text{eV} \end{aligned}$$

(d)

$$\begin{aligned} E &= hf \\ &= \frac{hc}{\lambda} \\ &= \frac{6.63 \times 10^{-34} \times 3.00 \times 10^{8}}{400 \times 10^{-9}} \\ &= 4.97 \times 10^{-19}\ \text{J} \\ &= \frac{4.97 \times 10^{-19}}{1.60 \times 10^{-19}} \\ &= 3.11\ \text{eV} \end{aligned}$$

16. (a)

$$E = hf - W$$

At the threshold frequency, E is 0, hence

$$\begin{aligned} hf &= W \\ f &= \frac{W}{h} \\ &= \frac{2.9 \times 1.60 \times 10^{-19}}{6.63 \times 10^{-34}} \\ &= 7.00 \times 10^{14}\ \text{Hz} \end{aligned}$$

$$\begin{aligned} \lambda &= \frac{c}{f} \\ &= \frac{3.00 \times 10^{8}}{7.00 \times 10^{14}} \\ &= 429\ \text{nm} \end{aligned}$$

The colour violet (or blue)

(b)

$$f = \frac{W}{h}$$
$$= \frac{4.25 \times 1.60 \times 10^{-19}}{6.63 \times 10^{-34}}$$
$$= 1.03 \times 10^{15}\ \text{Hz}$$

$$\lambda = \frac{c}{f}$$
$$= \frac{3.00 \times 10^{8}}{1.03 \times 10^{15}}$$
$$= 291\ \text{nm}$$

This is in the ultraviolet.

(c)

$$f = \frac{W}{h}$$
$$= \frac{4.42 \times 1.60 \times 10^{-19}}{6.63 \times 10^{-34}}$$
$$= 1.07 \times 10^{15}\ \text{Hz}$$

$$\lambda = \frac{c}{f}$$
$$= \frac{3.00 \times 10^{8}}{1.07 \times 10^{15}}$$
$$= 280\ \text{nm}$$

This is in the ultraviolet.

(d)

$$f = \frac{W}{h}$$
$$= \frac{5.1 \times 1.60 \times 10^{-19}}{6.63 \times 10^{-34}}$$
$$= 1.23 \times 10^{15}\ \text{Hz}$$

$$\lambda = \frac{c}{f}$$
$$= \frac{3.00 \times 10^{8}}{1.23 \times 10^{15}}$$
$$= 244\ \text{nm}$$

This is in the ultraviolet.

17. (a)

$$f = \frac{c}{\lambda} = \frac{3.00 \times 10^{8}}{390 \times 10^{-9}} = 7.69 \times 10^{14}\ \text{Hz}$$

$$f = \frac{c}{\lambda} = \frac{3.00 \times 10^{8}}{420 \times 10^{-9}} = 7.14 \times 10^{14}\ \text{Hz}$$

$$f = \frac{c}{\lambda} = \frac{3.00 \times 10^{8}}{460 \times 10^{-9}} = 6.52 \times 10^{14}\ \text{Hz}$$

$$f = \frac{c}{\lambda} = \frac{3.00 \times 10^{8}}{490 \times 10^{-9}} = 612 \times 10^{14}\ \text{Hz}$$

$$f = \frac{c}{\lambda} = \frac{3.00 \times 10^{8}}{520 \times 10^{-9}} = 577 \times 10^{14}\ \text{Hz}$$

(b)

$$E = qV$$
$$= 1.60 \times 10^{-19} \times 0.94$$
$$= 1.50 \times 10^{-19} \text{ J}$$

$$E = qV$$
$$= 1.60 \times 10^{-19} \times 0.71$$
$$= 1.14 \times 10^{-19} \text{ J}$$

$$E = qV$$
$$= 1.60 \times 10^{-19} \times 0.46$$
$$= 7.36 \times 10^{-20} \text{ J}$$

$$E = qV$$
$$= 1.60 \times 10^{-19} \times 0.30$$
$$= 4.80 \times 10^{-20} \text{ J}$$

$$E = qV$$
$$= 1.60 \times 10^{-19} \times 0.15$$
$$= 2.40 \times 10^{-20} \text{ J}$$

(d) (i) Threshold frequency is the intercept on the frequency axis, 5×10^{14} Hz.

(ii) Planck's constant is the gradient of graph, 6.5 J s.

(iii) Work function is the intercept on energy axis

$$= 3.53 \times 10^{-19} \text{ J}$$
$$= \frac{3.53 \times 10^{-19}}{1.60 \times 10^{-19}}$$
$$= 2.2 \text{ eV}$$

18. As $\lambda = \frac{h}{mv}$, the slower electron will have the longer wavelength.

19. (a) Its new wavelength will be half its original wavelength.

(b) Again, its new wavelength will be half its original wavelength. However, at this velocity, the wavelengths are 1/1000 of the values in part (a), and the change in wavelength is correspondingly smaller.

20. (a)

$$\lambda = \frac{\text{h}}{mv}$$
$$= \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times 3.00 \times 10^{7}}$$
$$= 2.43 \times 10^{-11} \text{ m}$$

(b)

$$\lambda = \frac{h}{mv}$$

$$v = \frac{h}{m\lambda}$$

$$= \frac{6.63 \times 10^{-34}}{0.1 \times 2.43 \times 10^{-11}}$$

$$= 2.73 \times 10^{-22} \text{ m s}^{-1}$$

21. (a) $E = \frac{1}{2} mv^2$

$$v = \sqrt{\frac{2E}{m}}$$

$$\lambda = \frac{h}{mv}$$

$$= \frac{h}{m\sqrt{\frac{2E}{m}}}$$

$$= \sqrt{\frac{h^2}{2mE}}$$

$$= \sqrt{\frac{(6.63 \times 10^{-34})^2}{2 \times 1.675 \times 10^{-27} \times 1.602 \times 10^{-13}}}$$

$$= \sqrt{8.19 \times 10^{-28}}$$

$$= 2.86 \times 10^{-14} \text{ m}$$

(b)

$$\lambda = \frac{h}{mv}$$

$$= \sqrt{\frac{h^2}{2mE}}$$

$$= \sqrt{\frac{(6.63 \times 10^{-34})^2}{2 \times 1.675 \times 10^{-27} \times 0.02 \times 1.602 \times 10^{-13}}}$$

$$= \sqrt{4.10 \times 10^{-26}}$$

$$= 2.02 \times 10^{-13} \text{ m}$$

22. (a)

$$\frac{1}{\lambda} = R_H\left(\frac{1}{n_f^{\ 2}} - \frac{1}{n_i^{\ 2}}\right)$$

$$= 1.097 \times 10^7 \left(\frac{1}{1^2} - \frac{1}{5^2}\right)$$

$$= 1.053 \times 10^7$$

$$\lambda = 9.496 \times 10^{-8} \text{ m}$$

(b)

$$\frac{1}{\lambda} = R_{\mathrm{H}}\left(\frac{1}{n_f^{\ 2}} - \frac{1}{n_i^{\ 2}}\right)$$
$$= 1.097 \times 10^7 \left(\frac{1}{2^2} - \frac{1}{5^2}\right)$$
$$= 2.304 \times 10^7$$
$$\lambda = 4.341 \times 10^{-7} \text{ m}$$

(c)

$$\frac{1}{\lambda} = R_{\mathrm{H}}\left(\frac{1}{n_f^{\ 2}} - \frac{1}{n_i^{\ 2}}\right)$$
$$= 1.097 \times 10^7 \left(\frac{1}{3^2} - \frac{1}{5^2}\right)$$
$$= 7.801 \times 10^5$$
$$\lambda = 1.282 \times 10^{-6} \text{ m}$$

23. (a) The transitions in the Balmer series are all into the state $n = 2$.

From $n = 8$ to $n = 2$:

$$\frac{1}{\lambda} = R_{\mathrm{H}}\left(\frac{1}{n_f^{\ 2}} - \frac{1}{n_i^{\ 2}}\right)$$
$$= 1.097 \times 10^7 \left(\frac{1}{2^2} - \frac{1}{8^2}\right)$$
$$= 2.571 \times 10^6$$
$$\lambda = 3.889 \times 10^{-7} \text{ m}$$

From $n = 10$ to $n = 2$:

$$\frac{1}{\lambda} = R_{\mathrm{H}}\left(\frac{1}{n_f^{\ 2}} - \frac{1}{n_i^{\ 2}}\right)$$
$$= 1.097 \times 10^7 \left(\frac{1}{2^2} - \frac{1}{10^2}\right)$$
$$= 2.633 \times 10^6$$
$$\lambda = 3.798 \times 10^{-7} \text{ m}$$

From $n = 12$ to $n = 2$:

$$\frac{1}{\lambda} = R_{\mathrm{H}}\left(\frac{1}{n_f^{\ 2}} - \frac{1}{n_i^{\ 2}}\right)$$
$$= 1.097 \times 10^7 \left(\frac{1}{2^2} - \frac{1}{12^2}\right)$$
$$= 2.666 \times 10^6$$
$$\lambda = 3.750 \times 10^{-7} \text{ m}$$

(b) As n increases, the wavelengths become smaller (higher energy) and the gaps between the wavelengths also become smaller. (The separation of the spectral lines becomes smaller as n increases.)

24. (a) The first two lines of the Lyman series correspond to jumps from the states E_2 and E_3 to E_1.

For transition from E_2 to E_1:

$$\begin{aligned}\Delta E &= (-3.40) - (-13.6) \\ &= 10.2 \text{ eV} \\ &= 10.2 \times 1.602 \times 10^{-19} \text{ J}\end{aligned}$$

$$\begin{aligned}E &= \mathrm{h} f \\ &= \mathrm{h}\frac{\mathrm{c}}{\lambda} \\ \lambda &= \frac{\mathrm{hc}}{E} \\ &= \frac{6.626 \times 10^{-34} \times 3.00 \times 10^{8}}{10.2 \times 1.602 \times 10^{-19}} \\ &= 1.22 \times 10^{-7} \text{ m}\end{aligned}$$

For transition from E_2 to E_1:

$$\begin{aligned}\Delta E &= (-1.51) - (-13.6) \\ &= 12.1 \text{ eV} \\ &= 12.1 \times 1.602 \times 10^{-19} \text{ J}\end{aligned}$$

$$\begin{aligned}E &= \mathrm{h} f \\ &= \mathrm{h}\frac{\mathrm{c}}{\lambda} \\ \lambda &= \frac{\mathrm{hc}}{E} \\ &= \frac{6.626 \times 10^{-34} \times 3.00 \times 10^{8}}{12.1 \times 1.602 \times 10^{-19}} \\ &= 1.03 \times 10^{-7} \text{ m}\end{aligned}$$

(b) The first two lines of the Balmer series correspond to jumps from the states E_3 and E_4 to E_2.

For transition from E_3 to E_2:

$$\Delta E = (-1.51) - (-3.40)$$
$$= 1.89 \text{ eV}$$
$$= 1.89 \times 1.602 \times 10^{-19} \text{ J}$$

$$E = hf$$
$$= h\frac{c}{\lambda}$$
$$\lambda = \frac{hc}{E}$$
$$= \frac{6.626 \times 10^{-34} \times 3.00 \times 10^{8}}{1.89 \times 1.602 \times 10^{-19}}$$
$$= 6.57 \times 10^{-7} \text{ m}$$

For transition from E_4 to E_2:

$$\Delta E = (-0.85) - (-3.40)$$
$$= 2.55 \text{ eV}$$
$$= 2.55 \times 1.602 \times 10^{-19} \text{ J}$$

$$E = hf$$
$$= h\frac{c}{\lambda}$$
$$\lambda = \frac{hc}{E}$$
$$= \frac{6.626 \times 10^{-34} \times 3.00 \times 10^{8}}{2.55 \times 1.602 \times 10^{-19}}$$
$$= 4.87 \times 10^{-7} \text{ m}$$

(c) The first two lines of the Paschen series correspond to jumps from the states E_4 and E_5 to E_3.

For transition from E_4 to E_3:

$$\Delta E = (-0.85) - (-1.51)$$
$$= 0.66 \text{ eV}$$
$$= 0.66 \times 1.602 \times 10^{-19} \text{ J}$$

$$E = \text{hf}$$
$$= \text{h}\frac{\text{c}}{\lambda}$$
$$\lambda = \frac{\text{hc}}{E}$$
$$= \frac{6.626 \times 10^{-34} \times 3.00 \times 10^{8}}{0.66 \times 1.602 \times 10^{-19}}$$
$$= 1.88 \times 10^{-6} \text{ m}$$

For transition from E_5 to E_3:

$$\Delta E = (-0.54) - (-1.51)$$
$$= 0.97 \text{ eV}$$
$$= 0.97 \times 1.602 \times 10^{-19} \text{ J}$$

$$E = \text{hf}$$
$$= \text{h}\frac{\text{c}}{\lambda}$$
$$\lambda = \frac{\text{hc}}{E}$$
$$= \frac{6.626 \times 10^{-34} \times 3.00 \times 10^{8}}{0.97 \times 1.602 \times 10^{-19}}$$
$$= 1.28 \times 10^{-6} \text{ m}$$

25. (a) The value of n_i would be infinite, therefore $\frac{1}{n_i^2}$ would be equal to zero.

(b) The series limit of the Lyman series corresponds to an energy of 13.6 eV.

$$\lambda = \frac{\text{hc}}{E}$$
$$= \frac{6.626 \times 10^{-34} \times 3.00 \times 10^{8}}{13.6 \times 1.602 \times 10^{-19}}$$
$$= 9.12 \times 10^{-8} \text{ m}$$

The series limit of the Balmer series corresponds to an energy of 3.40 eV.

$$\lambda = \frac{hc}{E}$$

$$= \frac{6.626 \times 10^{-34} \times 3.00 \times 10^{8}}{3.40 \times 1.602 \times 10^{-19}}$$

$$= 3.65 \times 10^{-7} \text{ m}$$

The series limit of the Paschen series corresponds to an energy of 1.51 eV.

$$\lambda = \frac{hc}{E}$$

$$= \frac{6.626 \times 10^{-34} \times 3.00 \times 10^{8}}{1.51 \times 1.602 \times 10^{-19}}$$

$$= 8.22 \times 10^{-7} \text{ m}$$

(c) The photon would have an energy of 13.6 eV.

26. (a) (i)

$$E = E_3 - E_2$$

$$= (-3.7) - (-5.5)$$

$$= 1.8 \text{ eV}$$

(ii)

$$E = hf$$

$$f = \frac{E}{h}$$

$$= \frac{1.8 \times 1.602 \times 10^{-19}}{6.626 \times 10^{-34}}$$

$$= 4.35 \times 10^{14} \text{ Hz}$$

$$\lambda = \frac{c}{f}$$

$$= \frac{3.00 \times 10^{8}}{4.35 \times 10^{14}}$$

$$= 6.89 \times 10^{-7} \text{ m}$$

(b)

$$E = E_4 - E_1$$
$$= (-1.6) - (-10.4)$$
$$= 8.8\,\text{eV}$$

$$\lambda = \frac{hc}{E}$$
$$= \frac{6.626 \times 10^{-34} \times 3.00 \times 10^{8}}{8.8 \times 1.602 \times 10^{-19}}$$
$$= 1.41 \times 10^{-7}\ \text{m}$$

27. Faintest detectable intensity $= 1.5 \times 10^{-11}$ watts m^{-2}

This corresponds to $= 1.5 \times 10^{-11} \times \pi \times (3.5 \times 10^{-3})^2$

$= 5.8 \times 10^{-16}$ joules per second falling on the pupil of the eye.

The energy of an individual photon of wavelength 700 nm is

$$E = hf$$
$$= \frac{hc}{\lambda}$$
$$= \frac{6.63 \times 10^{-34} \times 3.00 \times 10^{8}}{700 \times 10^{-9}}$$
$$= 2.84 \times 10^{-19}\ \text{J}$$

The number of photons required to produce 5.8×10^{-16} J

$$n = \frac{5.8 \times 10^{-16}}{2.84 \times 10^{-19}} = 2.04 \times 10^{3}$$

So approximately 2000 photons of wavelength 700 nm would have to fall on an eye for the eye to be just capable of detecting the light.